

7.1

Investigate Properties of Similar Triangles

Geometric shapes are often used in construction and design. Not only are certain shapes naturally pleasing to the eye, but they are also often useful for their structural properties.

Consider the shed shown. What geometric shapes can you recognize? Identify at least one pair of

- **congruent figures**
- **similar figures**



congruent figures

- identical shapes
- same size

similar figures

- identical shapes
- different sizes

Tools

- copy of shed drawing
- ruler
- protractor

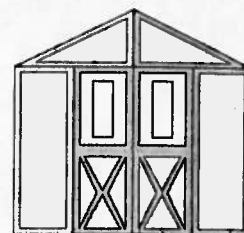
Optional

- tracing paper

Investigate

How can I recognize similar and congruent figures?

Look at the front face of the shed.

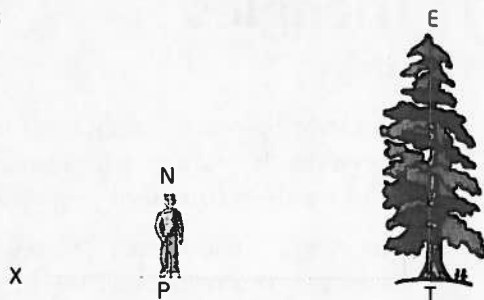


1. a) What pairs of congruent figures can you find?
b) Copy two or three pairs of congruent figures from the drawing of the shed. Label the vertices of each figure. What special properties do the corresponding sides and angles have?
2. *Reflect* Summarize the properties of congruent triangles.
3. a) What pairs of similar, but not congruent, figures can you find?
b) Make copies of two or three pairs of similar figures from the drawing of the shed. Label the vertices of each figure. What special properties do the corresponding sides and angles have?
4. *Reflect*
 - a) What is true about the corresponding angles in two congruent figures? in two similar figures?
 - b) What is true about the corresponding side lengths in two congruent figures? in two similar figures?
 - c) Copy and complete the table to summarize your answers to parts a) and b).

	Corresponding Angles Are	Corresponding Side Lengths Are ...
Congruent Figures		
Similar Figures		

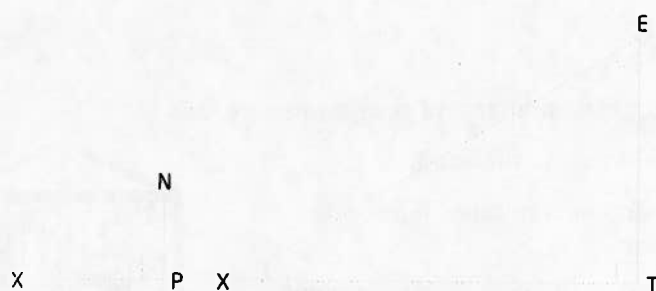
Example 1 Use Angles to Show That Two Triangles Are Similar

Identify a pair of similar triangles and explain why they are similar. Support each statement with a reason.



Solution

There are two overlapping similar triangles: $\triangle XPN$ and $\triangle XTE$. The diagram shows them separated:



You can show that these two triangles are similar by identifying three pairs of equal angles.

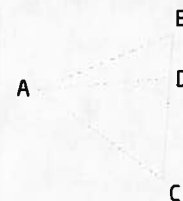
You can see three pairs of equal angles.

Statement	Reason
$\angle XPN = \angle XTE$	Both are 90° .
$\angle PXN = \angle TXE$	These are the same angle.
$\angle XNP = \angle XET$	The sum of the angles in any triangle is 180° .
$\triangle XPN \sim \triangle XTE$	All three pairs of angles are equal.

The third pair of angles in any two triangles is equal if the other two pairs are equal. So, it is only necessary to have two pairs of equal corresponding angles to show that two triangles are similar.

Connections

In $\triangle ABC$, the interior angle at vertex B can be called $\angle ABC$, $\angle CBA$, or $\angle B$. In the first two cases, the middle letter corresponds to the vertex of the angle.



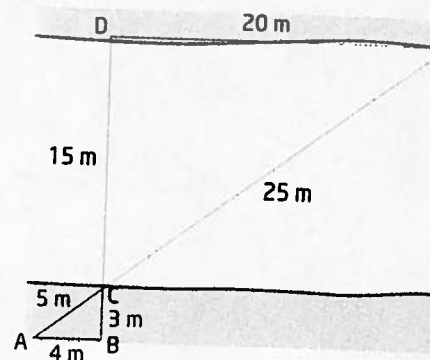
The three-letter system is useful to describe the three interior angles at vertex A: $\angle BAD$, $\angle DAC$, and $\angle BAC$. Referring simply to $\angle A$ can lead to confusion.

Connections

The symbol \sim means "is similar to." When you write a similarity statement, the order of the vertices must correctly identify pairs of equal angles and pairs of corresponding sides. In Example 1, $\triangle XPN \sim \triangle XTE$ is correct because the vertices correctly indicate pairs of equal angles and pairs of corresponding sides.

Example 2 Use Sides to Show That Two Triangles Are Similar

Are the two triangles in the diagram similar? Explain your reasoning.



Solution

You can show that two triangles are similar by showing that three pairs of corresponding sides are proportional.

The two triangles are $\triangle EDC$ and $\triangle ABC$.

$$\begin{aligned} \frac{ED}{AB} &= \frac{20}{4} & \frac{CE}{CA} &= \frac{25}{5} & \frac{DC}{BC} &= \frac{15}{3} \\ &= \frac{5}{1} & &= \frac{5}{1} & &= \frac{5}{1} \end{aligned}$$

The three pairs of corresponding sides are all in the ratio 5:1. Therefore, $\triangle EDC \sim \triangle ABC$.

Since the triangles in Example 2 are similar, their corresponding angles are equal. That is, $\angle CAB = \angle CED$, $\angle CBA = \angle CDE$, and $\angle ACB = \angle ECD$.

Key Concepts

- Congruent figures have the same size and shape. Similar figures have the same shape, but different sizes.

- Similar triangles have the following properties:

– Corresponding angles

are equal:

$$\angle A = \angle T$$

$$\angle B = \angle U$$

$$\angle C = \angle V$$

– Ratios of corresponding sides are equal:

$$\frac{AB}{TU} = \frac{BC}{UV} = \frac{CA}{VT}, \text{ or, in ratio notation, } AB:TU = BC:UV = CA:VT$$

